

# Mathematics Standard Articulated by Grade Level

## INTRODUCTION

The Arizona Mathematics Standard Articulated by Grade Level describes a connected body of mathematical understandings and competencies that provide a foundation for all students. This standard is coherent, focused on important mathematics, and well articulated across the grades. Concepts and skills that are critical to the understanding of important processes and relationships are emphasized.

The need to understand and use a variety of mathematical strategies in multiple contextual situations has never been greater. Utilization of mathematics continues to increase in all aspects of everyday life, as a part of cultural heritage, in the workplace, and in scientific and technical communities. Today's changing world will offer enhanced opportunities and options for those who thoroughly understand mathematics.

Communication, problem solving, reasoning and proof, connections, and representation are the process standards as described in the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM). These process standards are interwoven within each of the content strands of the Arizona Mathematics Standard and are explicitly connected to the teaching of specific performance objectives in the grade level documents. The process standards emphasize ways to acquire and apply the content knowledge.

Mathematics education should enable students to fulfill personal ambitions and career goals in an informational age. In the NCTM *Principles and Standards* document it asks us to “*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it*”.<sup>1</sup> The Arizona Mathematics Standard Articulated by Grade Level is intended to facilitate this vision.

## BACKGROUND

The State Board of Education adopted the Mathematics Standard Articulated by Grade Level in 2003 to define what Arizona students need to know and be able to do at each grade level through the end of tenth grade. Developed by a committee comprised of a diverse group of educators, this standard was written in response to the requirements of *No Child Left Behind Act of 2001* (NCLB).

## RATIONALE

In 2007 the State Board of Education began the process for increasing the high school graduation requirement in mathematics from two to four years. This requirement was

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<sup>1</sup> National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics*, NCTM Publications, Reston, VA, 2000, p. 3.

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approved in December 2007 effective with the graduating class of 2013. This increase, along with the need to complete a periodic review of the standard, prompted the Arizona Department of Education to initiate the process of refining and rearticulating the Mathematics Standard. This refinement and articulation project began in June 2007 and was completed in June 2008.

## METHODOLOGY

Work teams representing populations from around the state were formed. These groupings were comprised of large and small schools, rural and urban schools, and were ethnically diverse. Included were classroom teachers, curriculum directors, mathematics teacher leaders, Career and Technical Education teachers, second-career teachers, and university/community college faculty. The goal was to revise and articulate the Mathematics Standard K-12 to align with the increased state requirement of four years of high school mathematics.

The mathematics revision teams utilized the National Council of Teachers of Mathematics *Principles and Standards* as a reference in the development of the revised Mathematics Standard. Additionally, the findings and recommendations from the National Mathematics Advisory Panel, the American Diploma Project Benchmarks, the National Assessment of Educational Progress Framework, the Curriculum Focal Points, the Framework for 21<sup>st</sup> Century Skills, and other states' frameworks were used as guiding documents.

The revision grade level teams created draft documents with performance objectives articulated to the appropriate grade levels. Over a period of months, these teams and smaller sub-committees of teams refined the draft documents based on clarity, cohesiveness, and comprehensiveness. Reasonableness, usefulness, and appropriateness were key guidelines for the articulation process. The measurability of each performance objective was also a consideration.

External reviews by nationally recognized consultants brought a broader perspective to the refinement process. Another important step in the process was the gathering of public comment. In March 2008, drafts of the Revised Mathematics Standard Articulated by Grade Level, along with a survey to gather feedback, were posted on the Arizona Department of Education website. This provided the public with easy access to the documents, and a survey allowed reviewers a means for submitting comments. Also, crosswalks were created from the Draft 2008 Mathematics Standard to the 2003 Mathematics Standard and were posted on the website. The public had the opportunity to submit comments and suggestions, either electronically or in writing, until the survey closing date of March 28, 2008. Additionally, five public hearings were held in March throughout the state offering further opportunities for public feedback.

After all the public comments were collected, organized, and categorized by grade level and topic, the revision teams met to determine what modifications to the standard document would be appropriate. Upon completion of the revision work, crosswalks were created to assist educators with the transition

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from the 2003 Arizona Mathematics Standard Articulated by Grade Level to the revised 2008 Mathematics Standard.

## ORGANIZATION OF THE MATHEMATICS STANDARD

The Mathematics Standard Articulated by Grade Level is divided into five main strands:

- Number and Operations
- Data Analysis, Probability, and Discrete Mathematics
- Patterns, Algebra, and Functions
- Geometry and Measurement
- Structure and Logic.

Each strand is divided into concepts that broadly define the skills and knowledge that students are expected to know and be able to do. Under each concept are performance objectives (POs) that more specifically delineate the ideas to be taught and learned.

The comprehensive document (K-12) is designed so that teachers can read the performance objectives across grade levels to incorporate learning from previous, current, and future grade levels. The standard is separated into two separate documents due to the addition of College Work Readiness (grades 11-12). The first document spans grade levels K through 6, and the second document covers grades 7 through College Work Readiness. Viewing the Mathematics Standard document from left to right helps the teacher to see the mathematics continuum across the grade levels. There is a purposeful clustering of performance objectives in order to emphasize certain key understandings. Every effort was made

to eliminate repetitions. The intent was to build on the learning in previous grade levels, connect important ideas, and highlight new content each year. This coherency supports students in developing new understandings and skills. Looking down each individual column enables a teacher to see the performance objectives that students are expected to know and be able to do at any grade level.

This organization does not imply that the teaching and learning of mathematics should be fragmented or compartmentalized. Mathematics is a highly interconnected discipline; important mathematical ideas from all five mathematics strands need to be continuously integrated as needed to make meaning and connections to other concepts and performance objectives. In each grade level document, these connections are highlighted.

The order of the strands, concepts, and performance objectives (POs) in the Mathematics Standard document are not intended to be a checklist for mathematics instruction. Mathematical concepts develop with a spiraling of ideas/skills that are interconnected and dependent on each other, and this is reflected in the standard document. Effective instruction often incorporates several performance objectives into an integrated experience of learning for the student.

The content in College Work Readiness (grades 11-12) is a new addition to the Mathematics Standard. This content is separated into the five main strands. Performance objectives highlighted in italics in the document have been identified as core to an Algebra II course. As districts/schools create additional high school mathematics courses, they may select

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from the comprehensive set of performance objectives contained within the five strands.

New to the 2008 Mathematics Standard is the development of more comprehensive grade level documents. The format of these documents will support the implementation of the revised standard. After each concept statement, there are summary expectations appropriate for that specific grade level. These statements provide a roadmap for instruction. Teachers will notice that there are now three columns of information. The first column lists the performance objectives with accompanying strand/concept and content area connections. The middle column highlights explicit connections to Strand 5, Concept 2 performance objectives. These performance objectives are grounded in the core processes of logic, reasoning, problem-solving and proof. The third column provides instructional support to teachers in the form of explanation and examples.

## **Strand One: Number and Operations**

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers such as, whole numbers, integers, fractions, and decimals and the relationships between them and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet.

Students develop a sense of what numbers are, i.e., to use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

### **Concept 1: Number Sense**

Understand and apply numbers, ways of representing numbers, the relationships among numbers, and different number systems.

### **Concept 2: Numerical Operations**

Understand and apply numerical operations and their relationship to one another.

### **Concept 3: Estimation**

Use estimation strategies reasonably and fluently while integrating content from each of the other strands.

## **Strand 2: Data Analysis, Probability, and Discrete Mathematics**

This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares students for the study of discrete functions as well as to make valid inferences, decisions, and arguments.

Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are

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used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

## Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization, and representation to analyze and sort data. This is considered to be the analysis and interpretation of numerical data in terms of samples and populations.

## Concept 2: Probability

Understand and apply the basic concepts of probability. This is the field of mathematics that deals with the likelihood that an event will occur expressed as the ratio of the number of favorable outcomes in the set of outcomes to the total number of possible outcomes.

## Concept 3: Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes. This field of mathematics is generally referred to as Combinatorics.

## Concept 4: Vertex-Edge Graphs

Understand and apply the concepts of vertex-edge graphs and networks. This field connects graph theory with practical problems.

## Strand 3: Patterns, Algebra, and Functions

Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and

graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change.

Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

## Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically. Students begin with simple repetitive patterns of many iterations. This is the beginning of recursive thinking. Later, students can study sequences that can best be defined using recursion.

## Concept 2: Functions & Relationships

Describe and model functions and their relationships. For example, distribution and communication networks, laws of physics, population models, and statistical results can all be represented in the symbolic language of algebra.

## Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations. Algebraic representation is about abstract structures and about using the principles of those structures in solving problems expressed with symbols.

## Concept 4: Analysis of Change

Analyze how changing the values of one quantity corresponds to change in the values of another quantity.

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## Strand 4: Geometry and Measurement

Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world.

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. A major emphasis in this strand is becoming familiar with the units and processes that are used in measuring attributes.

### Concept 1: Geometric Properties

Analyze the attributes and properties of two- and three-dimensional figures and develop mathematical arguments about their relationships (in conjunction with strand 5, concept 2).

### Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

### Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

### Concept 4: Measurement

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

## Strand 5: Structure and Logic

This strand emphasizes the core processes of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning (both inductive and deductive) as they make conjectures and test the validity of arguments and proofs. Concept two develops the core processes as students evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

### Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems. Determine step-by-step series of instructions to explain mathematical processes.

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## **Concept 2: Logic, Reasoning, Problem Solving, and Proof**

Evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications. Develop mathematical arguments based on induction and deduction, and distinguish between valid and invalid arguments.